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Some More Results on Stolarsky-3 Mean Labeling of Graphs

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Abstract- In this paper, we contribute some new results for Stolarsky-3 Mean labeling of graphs. We have already proved Stolarsky-3 Mean labeling of some standard Graphs. We use some more standard graphs to derive the results of Stolarsky-3 Mean labeling. We prove Stolarsky-3 Mean labeling of Bistar, Planar Grid, Crown and Square of a Path.

***Keywords*:** Graph Labeling, Mean Labeling, Stolarsky-3 Mean Labeling, Bistar, Crown.

I. INTRODUCTION

The graph G = (V, E) is considered here will be finite, simple and undirected. We follow Gallian **[1]** for all detailed survey of graph labeling and we refer Harary **[2]** for all other standard terminologies and notations. The concept of "Mean **Labeling"**has been introduced by S.Somasundaram and R.Ponraj in 2004**[3**] and S.Somasundaram and S.S. Sandhya introduced the concept of "**Harmonic Mean Labeling of graphs" in [4].**The concept of **"Stolarsky-3 Mean Labeling**" has introduced by S.S.Sandhya, E.Ebin Raja Merely and S.Kavitha in **[6].**

We will give the following definitions and other information's are useful for our present investigation.

Definition 1.1: A graph G with p vertices and q edges is said to be S be Stolarsky-3 Mean graph if each vertex x∈v is assigned distinct labels f(x) from 1,2,….,q+1 and each edge e=uv isassigned the distinct labels f(e=uv) = $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ (or) $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G.

Definition 1.2: The Bistar $B_{m,n}$ is the graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2. The edge of K_2 is called the central edge of $B_{m,n}$ and the vertices of K_2 are called the central vertices of $B_{m,n}$.

Definition 1.3: The Square G^2 of a graph G has V (G^2) = V (G) with u, v adjacent in G^2 whenever d(u,v) ≤ 2 in G

Definition 1.4: The Corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G (which has p vertices and p copies of G_2) and then joining the ith vertex of G_1 to every vertices in the ith copy of G_2. Here we restrict ourselves to corona with cycles. The graph $C_n \odot K_1$ is called a Crown.

Definition 1.5: The Cartesian product of two graphs $G_1$and G_2 is thee graph $G_1 \times G_2$ with the vertex set $V_1 \times V_2$ and two vertices u =(u_1,u_2) and v =(v_1, v_2) are adjacent whenever u_1 =v_1 and u_2 adjacent to v_2 or u_2 =v_2 and u_1 adjacent to v_1.

Definition 1.6:$P_m \times P_n$ is called a planar grid.

II. MAIN RESULTS

Theorem 2.1: The Bistar $B_{m,n}$ is Stolarsky-3 Mean graph if m≤7 and n≤10.

Proof: Let $B_{m,n}$ be a Bistar graph.
Consider two cases.
Case (i) m≤ 7& n ≤ 10
Let u, v,u_i, v_j, 1≤ $i \le m$, 1≤ j ≤ n are the vertices of $B_{m,n}$ and uv, uu_i, vv_j
1≤ $i \le m$, 1≤ $j \le n$ are the edges of $B_{m,n}$.
Define a function **f** : V($B_{m,n}$) →{ 1,2,…., q+1} by
f(u) = 1 .
f(u_i) = 2i +1, 1 ≤ i ≤m.

$f(v_j) = 2n + 2(j-1)$, $1 \le j \le n$.
Then the edges are labeled with
$f(uv) = 1$,
$f(uu_i) = i + 1$, $1 \le i \le m$.
$f(vv_j) = f(uu_m) + j$, $1 \le j \le n$.
Then the edge labels are distinct.
Hence $B_{m,n}$ is Stolarsky-3 Mean graph if m≤7 and n≤10.

Example 2.2: The Stolarsky-3 Mean labeling of $B_{7,10}$ is given below

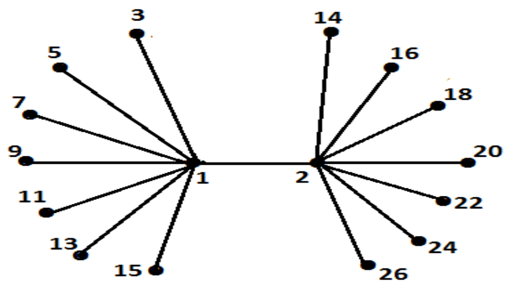

Figure:1

Case (ii) m≥ 8 , n≥11
Let u, v, u_i, v_j, $1 \le i \le m$, $1 \le j \le n$ are the vertices of $B_{m,n}$ and uv, uu_i, vv_j $1 \le i \le m$, $1 \le j \le n$ are the edges of $B_{m,n}$.

When m=8 and n=11
Let the label of the vertices are
u = 1.
$u_i = 2i+1$, $1 \le i \le 8$.
And the edges are labeled as
uv =1.
$uu_i = i + 1$, $1 \le i \le 7$.
$uu_8 = 10$.
Here the number 9 cannot be labeled
$vv_j = uu_8 + j$, $1 \le j \le 3$.
$vv_4 = 15$
$vv_j = vv_{j-1} + 1$, $5 \le j \le 10$.
$vv_{11} = 23$
Here the numbers 14 and 22 cannot be labeled
Hence $B_{m,n}$ is not a Stolarsky-3 Mean graph if m≥ 8 , n≥11.

Example 2.2: The Stolarsky-3 Mean labeling of $B_{8,11}$ is given below

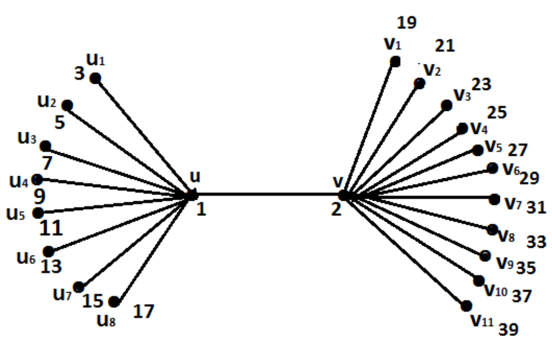

Figure: 2

Theorem 2.3: The graph P_n^2 is Stolarsky-3 Mean graph.

Proof: Let P_n be the path $u_1, u_2, \dots, u_n$.
Clearly P_n^2 has n vertices and 2n-3 edges.
Define a function $f : V(P_n^2) \to \{1,2,\dots, q+1\}$ by
$f(u_1) = 1$.
$f(u_2) = 2$.
$f(u_i) = 2i-3$, $3 \le i \le n$.
Then the edges are labeled as
$f(u_i u_{i+1}) = 2i-1$, $1 \le i \le n-1$.
$f(u_i u_{i+2}) = 2i$, $1 \le i \le n-2$.
Then the edge labels are distinct. Hence P_n^2 is Stolarsky-3 Mean graph.

Example 2.4: The Stolarsky-3 Mean labeling of P_6^2 is given below.

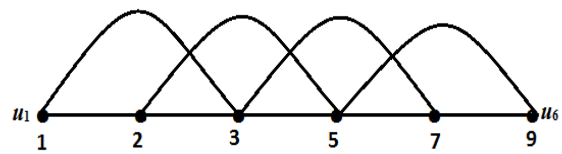

Figure: 3

Theorem 2.5: The Planar grid $P_m \times P_4$ is Stolarsky-3 Mean graph.

Proof: Let $V(P_m \times P_4) = \{a_{ij}, 1 \le i \le m, 1 \le j \le 4\}$
And $E(P_m \times P_4) = \{a_{i(j-1)} a_{ij}, 1 \le i \le m, 1 \le j \le 4 \cup a_{(i-1)j} a_{ij}, 2 \le i \le m, 1 \le j \le 4\}$
Define a function $f : V(P_m \times P_4) \to \{1,2,\dots, q+1\}$ by
$f(a_{1j}) = j$, $1 \le j \le 4$.
$f(a_{2j}) = f(a_{(i-1)4}) + 2 + j$, $2 \le i \le m$, $1 \le j \le 4$.
$f(a_{ij}) = f(a_{(i-1)4}) + 3 + j$, $3 \le i \le m$, $1 \le j \le 4$.
Then the edges are labeled as
$f(a_{ij} a_{ij+1}) = 7(i-1) + j$, $1 \le i \le m$, $1 \le j \le 3$.
$f(a_{ij} a_{i+1j}) = 3 + 7(i-1) + j$, $1 \le i \le m-1$, $1 \le j \le 3$.
Then we get distinct edge labels.
Hence f is Stolarsky-3 Mean labeling.

Example 2.6: The labeling pattern of $P_5 \times P_4$ is given below.

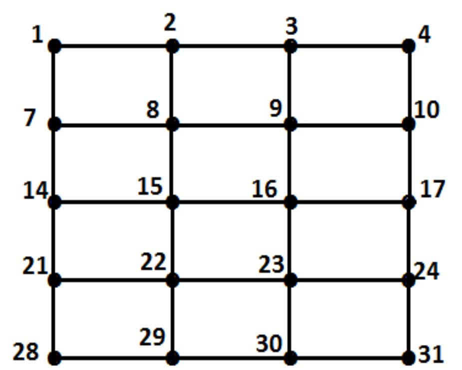

Figure: 4

Theorem 2.7: The Planar grid $P_m \times P_3$ is Stolarsky-3 Mean graph.

Proof: Let V($P_m \times P_3$) ={a_{ij} , 1≤ $i \le m$, $1 \le j \le 3$} and E($P_m \times P_3$) = {$a_{i(j-1)}a_{ij}$, 1≤ $i \le m$, 2≤ $j \le 3$ ∪ $a_{(i-1)j}a_{ij}$, 2≤ $i \le m$, 1≤ j≤3}
Define a function **f** : V($P_m \times P_3$)→{ 1,2,…., q+1} by
f(a_{1j}) = j , 1≤ $j \le 3$.
f(a_{ij})=**f**$(a_{(i-1)3})$+2+j, 2≤ $i \le m$, 1≤j≤3.
Then the edges are labeled as
f$(a_{ij}a_{ij+1})$=5(i -1)+j, 1≤ $i \le m$,1≤ j≤2.
f$(a_{ij}a_{i+1j})$=2+ 5(i-1)+j , 1≤ $i \le m-1$, 1≤ j≤3.
Then we get distinct edge labels.
Hence f is Stolarsky-3 Mean labeling.
Example 2.8: The labeling pattern of $P_6 \times P_3$ is given below.

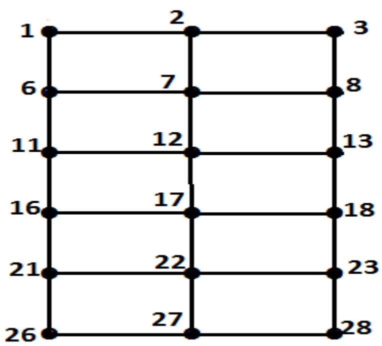

Figure: 5

Theorem 2.9: The Crown $C_n\boldsymbol{\Theta}K_1$ is a Stolarsky-3 Mean graph.
Proof: Let C_n be the cycle $u_1, u_2, \dots, u_n, u_1$ and let $v_1, v_2, \dots, v_n$, be the pendant vertices attached to u_i, 1≤ $i \le n$.
Let G = $C_n\boldsymbol{\Theta}K_1$
Define a function
f : V(G) →{ 1,2,…., q+1} by
f(u_i) =2i-1, 1≤ $i \le n$.
f(v_i) = 2i, 1≤ $i \le n$.
Then the edge labels are distinct.
Hence Crown isStolarsky-3 Mean labeling.
Example 2.10: TheStolarsky-3 Mean labeling of $C_6\boldsymbol{\Theta}K_1$ is given below.

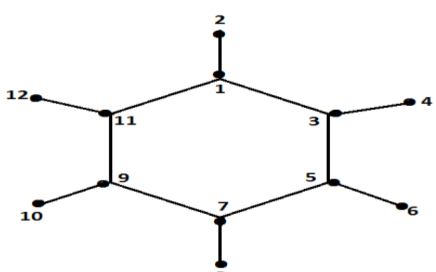

Figure 6

Theorem 2.11: The graph $C_n\boldsymbol{\Theta}K_{1,2}$ is Stolarsky-3 Mean graph.
Proof: Let C_n be the cycle $u_1, u_2, \dots, u_n, u_1$ and let v_i , w_i be the pendant vertices attached to u_i, 1≤ i≤n.
Let G = $C_n\boldsymbol{\Theta}K_{1,2}$
We define f: V(G)→ {1,2,3,….,q+1} by
f(u_i) = 3i-1, 1≤ $i \le n$.
f(v_i) = 3i-2, 1≤ $i \le n$.
f(w_i) = 3i, 1≤ $i \le n$.
Then the edge labels are distinct.
Hence$C_n\boldsymbol{\Theta}K_{1,2}$ is stolarsky-3 Mean graph.
Example 2.12: The Stolarsky-3 Mean labeling of $C_5\boldsymbol{\Theta}K_{1,2}$is given below.

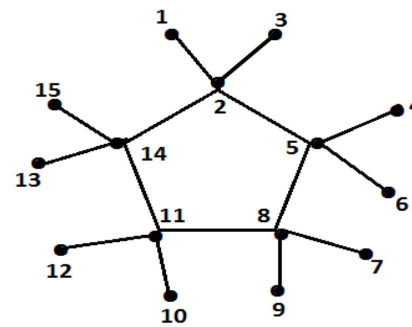

Figure 7

Theorem 2.13: The graph $C_n\boldsymbol{\Theta}K_{1,3}$ is Stolarsky-3 Mean graph.
Proof: Let C_n be the cycle $u_1, u_2, \dots, u_n, u_1$ and let K_3 be the cycle with the vertices $v_i, w_i x_i$attached to each of the vertices u_i, 1≤ i≤n.
Let G = $C_n\boldsymbol{\Theta}K_{1,3}$
We define f: V(G)→ {1,2,3,….,q+1} by
f(u_i) = 4i-2, 1≤ $i \le n$.
f(v_i) = 4i-3, 1≤ $i \le n$.
f(w_i) = 4i-1, 1≤ $i \le n$.
f(x_i) = 4i, 1≤ $i \le n$.
Then the edge labels are distinct.
Hencethe graph $C_n\boldsymbol{\Theta}K_{1,3}$ is stolarsky-3 Mean graph.
Example 2.14: The Stolarsky-3 Mean labeling of $C_5\boldsymbol{\Theta}K_{1,3}$is given below.

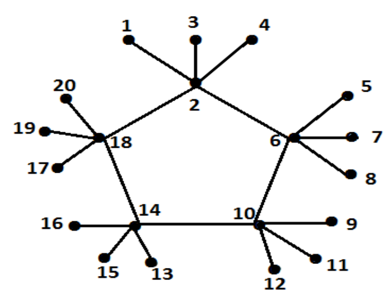

Figure 8

Theorem 2.15: The graph $C_n\boldsymbol{\Theta}K_3$ is Stolarsky-3 Mean graph.

Proof: Let C_n be the cycle $u_1, u_2, \dots, u_n, u_1$ and let K_3 be the cycle with the vertices v_i, w_i joining to each of the vertices u_i, $1 \le i \le n$.
Let G = $C_n \boldsymbol{\Theta} K_3$
We define f: V(G)→ {1,2,3,….,q+1} by
$\mathbf{f}(u_i)$ = 4i-2, $1 \le i \le n$.
$\mathbf{f}(v_i)$ = 4i-3, $1 \le i \le n$.
$\mathbf{f}(w_i)$ = 4i, $1 \le i \le n$.
Then the edge labels are distinct.
Hencethe graph $C_n \boldsymbol{\Theta} K_3$ is stolarsky-3 Mean graph.
Example 2.16: The Stolarsky-3 Mean labeling of $C_5 \boldsymbol{\Theta} K_3$ is given below.

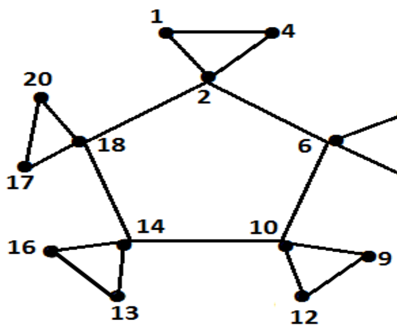

Figure 9

Theorem 2.17: The graph $P_n \boldsymbol{\Theta} K_3$ is Stolarsky-3 Mean graph.
Proof: Let P_n be the Path on n vertices $u_1, u_2, \dots, u_n$ and let K_3 be the cycle with the vertices v_i, w_i joining to each of the vertices u_i, $1 \le i \le n$.
Let G = $P_n \boldsymbol{\Theta} K_3$
We define f: V (G)→ {1, 2, 3,…., q+1} by
$\mathbf{f}(u_i)$ = 4i-3, $1 \le i \le n$.
$\mathbf{f}(v_i)$ = 4i-2, $1 \le i \le n$.
$\mathbf{f}(w_i)$ = 4i-1, $1 \le i \le n$.
Then the edges are labeled as
$\mathbf{f}(u_i u_{i+1})$ =4i, $1 \le i \le n-1$.
$\mathbf{f}(u_i v_i)$ =4i-3, $1 \le i \le n-1$.
$\mathbf{f}(u_i w_i)$ =4i-2, $1 \le i \le n-1$.
Hence the edge labels are distinct.
Hencethe graph $P_n \boldsymbol{\Theta} K_3$ is stolarsky-3 Mean graph.
Example 2.18 : The Stolarsky-3 Mean labeling of $P_5 \boldsymbol{\Theta} K_3$ is given below.

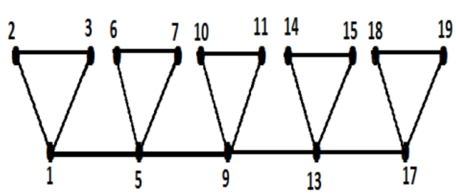

Figure 10

Theorem 2.19: The graph $P_n \boldsymbol{\Theta} K_{1,2}$ is Stolarsky-3 Mean graph.
Proof: Let P_n be the Path on n vertices $u_1, u_2, \dots, u_n$ and let v_i, w_i be the pendant vertices attached to each of the vertices u_i, $1 \le i \le n$.
Let G = $P_n \boldsymbol{\Theta} K_{1,2}$
We define f: V(G)→ {1,2,3,….,q+1} by
$\mathbf{f}(u_i)$ = 3i-2, $1 \le i \le n$.
$\mathbf{f}(v_i)$ = 3i-1, $1 \le i \le n$.
$\mathbf{f}(w_i)$ = 3i, $1 \le i \le n$.
Then the edges are labeled as
$\mathbf{f}(u_i u_{i+1})$ =3i, $1 \le i \le n-1$.
$\mathbf{f}(u_i v_i)$ =3i-2, $1 \le i \le n-1$.
$\mathbf{f}(u_i w_i)$ =3i-1, $1 \le i \le n-1$.
Then the edge labels are distinct.
Hencethe graph $P_n \boldsymbol{\Theta} K_{1,2}$ is stolarsky-3 Mean graph.
Example 2.20: The Stolarsky-3 Mean labeling of $P_5 \boldsymbol{\Theta} K_{1,2}$ is given below.

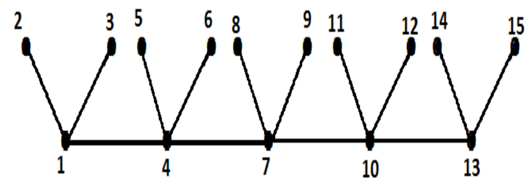

Figure 11

Theorem 2.21: The graph $P_n \boldsymbol{\Theta} K_{1,3}$ is Stolarsky-3 Mean graph.
Proof: Let P_n be the Path on n vertices$u_1, u_2, \dots, u_n$ and let v_i, w_i, x_i be the pendant vertices attached to each of the vertices u_i, $1 \le i \le n$.
Let G = $P_n \boldsymbol{\Theta} K_{1,3}$
We define f: V (G)→ {1, 2, 3,…., q+1} by
$\mathbf{f}(u_i)$ = 4i-3, $1 \le i \le n$.
$\mathbf{f}(v_i)$ = 4i-2, $1 \le i \le n$.
$\mathbf{f}(w_i)$ = 4i-1, $1 \le i \le n$.
Then the edges are labeled as
$\mathbf{f}(u_i u_{i+1})$ =4i, $1 \le i \le n-1$.
$\mathbf{f}(u_i v_i)$ =4i-3, $1 \le i \le n-1$.
$\mathbf{f}(u_i w_i)$ =4i-2, $1 \le i \le n-1$.
$\mathbf{f}(u_i x_i)$ =4i-1, $1 \le i \le n-1$.
Then the edge labels are distinct.
Hencethe graph $P_n \boldsymbol{\Theta} K_{1,3}$ is stolarsky-3 Mean graph.
Example 2.22: The Stolarsky-3 Mean labeling of $P_5 \boldsymbol{\Theta} K_{1,3}$ is given below.

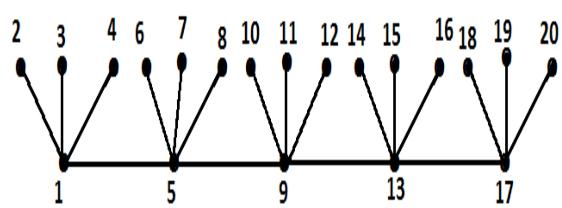

Figure 12

III. CONCLUSION

The Study of labeled graph is important due to its diversified applications. It is very interesting to investigateStolarsky-3 mean graphs which admit Stolarsky-3 Mean Labeling. The derived results are demonstrated by means of sufficient Illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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